Superconditioning

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Diaconis and Zabell (1982)

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Theorem. P' comes from P by conditionalization if and only if there exists a $B \geq 1$ such that

$$P'(\omega) \leq BP(\omega)$$

Takeaway

In principle, the shift from P to P' can be represented by conditionalizing on E.

The conditioning proposition E is an idealization that does not (necessarily) stand for something the agent can express.

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The conditioning proposition E is an idealization that does not (necessarily) stand for something the agent can express.

That a superconditioning space exists is a plausible **necessary condition** for the shift to be a **learning event**.

Plan

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- A shift from a prior to a set of posteriors. This provides insights about the reflection principle.
- A shift from a prior to two ore more distinct sets of posteriors. This speaks to the common prior assumption and a model of time-slice rationality.

First extension

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- there is a partition $E_1, \ldots, E_n \in \mathfrak{A}$ of Λ such that

$$P_i(\omega) = Q(E_\omega \mid E_i) \quad \text{if } Q(E_i) > 0.$$

Superconditioning

Theorem. The shift from P to $P_1, \ldots, P_n$ can be embedded in a conditioning model if and only if there exist constants $0 \leq B_1, \ldots, B_n \leq 1$, $\sum_i B_i = 1$ such that for all $\omega \in \Omega$,

$$P(\omega) = \sum_i B_i P_i(\omega).$$

Reflection

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In this case $B_i = P(A_i)$ and

$$P(\omega) = \sum_i P(A_i) P_i(\omega).$$

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Since the generalized reflection principle doesn't require priors over posteriors, it applies to (genuinely) diachronic shifts.

If we take being representable in a superconditioning space as a necessary condition for a shift to be a learning event, then the reflection principle is a necessary condition as well.

Second extension

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- there is a partition $E_{11}, \ldots, E_{1n} \in \mathfrak{A}$ of Λ such that

$$P_{1i}(\omega) = Q(E_\omega \mid E_{1i}) \text{ if } Q(E_{1i}) > 0;$$

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- there is a partition $E_{21}, \ldots, E_{2m} \in \mathfrak{A}$ of Λ such that

$$P_{2j}(\omega) = Q(E_\omega \mid E_{2j}) \text{ if } Q(E_{2j}) > 0.$$

Superconditioning

Theorem. The shifts from P to $P_{11}, \ldots, P_{1n}$ and $P_{21}, \ldots, P_{2m}$ can be embedded in a conditioning model if and only if there exist constants $0 \leq B_{11}, \ldots, B_{1n} \leq 1$ and $0 \leq B_{21}, \ldots, B_{2m} \leq 1$ such that $\sum_i B_{1i} = \sum_j B_{2j} = 1$ and for all $\omega \in \Omega$,

$$P(\omega) = \sum_i B_{1i} P_{1i}(\omega) = \sum_j B_{2j} P_{2j}(\omega).$$

Common prior

We say that $P_{11}, \ldots, P_{1n}$ and $P_{21}, \ldots, P_{2m}$ have a **common prior** if there is a P such that the shift from P to $P_{11}, \ldots, P_{1n}$ and $P_{21}, \ldots, P_{2m}$ can be embedded in a conditioning model.

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Corollary. $P_{11}, \ldots, P_{1n}$ and $P_{21}, \ldots, P_{2m}$ have a common prior if and only if the intersection of their convex spans is nonempty.

Game theory

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The result shows that for a common prior to exist the players' posteriors must agree to some extent.

Time-slice epistemology

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According to Williamson, your current credences are not constrained by your past credences, but by your current evidence. At any given time, your current credence in a proposition should match the prior conditional probability of that proposition, conditional on your current evidence. The prior probability distribution is a distinguished measure of "something like the intrinsic plausibility of hypotheses prior to investigation" [...], and your current evidence is just your current knowledge [..]. Since knowledge is not necessarily cumulative for rational agents, this proposal answers challenges involving rational memory loss. Since your current mental states include your current knowledge, this proposal advances time-slice epistemology.

Moss, *Time-slice rationality and action under indeterminacy*

Final extension

P_1 and P_2 **come from** P **by conditionalization** if there exists a probability space $(\Lambda, \mathfrak{A}, Q)$ such that

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- there is an $E_1 \in \mathfrak{A}$ such that $Q(E_1) > 0$ and $P_1(\omega) = Q(E_\omega \mid E_1)$;
- there is an $E_2 \in \mathfrak{A}$ such that $Q(E_2) > 0$ and $P_2(\omega) = Q(E_\omega \mid E_2)$.

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Theorem. P_1 and P_2 come from P by conditionalization if and only if there exists a $B \geq 1$ such that, for all $\omega \in \Omega$,

$$P_1(\omega),\ P_2(\omega) \leq BP(\omega).$$

Takeaway

This condition does not generate any interesting dynamic coherence between P_1 and P_2:

- $P_1(E) = 1$ and $P_2(E) = 0$;
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In the rich framework of sets of posteriors, the two time slices cannot be fully divorced from each other: an underlying prior exists only if their convex spans overlap.

Thank you!